Mayer (A. M.)

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# RESEARCHES IN ACOUSTICS.

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CONTAINING

1. An Experimental Confirmation of Fourier's Theorem as applied to the decomposition of the vibrations of a composite sonorous wave into its elementary pendulum-vibrations.
2. An Experimental Illustration of Helmholtz's Hypothesis of Audition.
3. Experiments on the supposed Auditory Apparatus of the Culex Mosquito.
4. Suggestions as to the function of the spiral scalæ of the Cochlea, leading to an Hypothesis of the Mechanism of Audition.
5. Seven Experimental Methods of Sonorous Analysis described and discussed.
6. The Curve of a Musical Note, formed by combining the sinusoids of its first six harmonics; and the curves formed by combining the curves corresponding to various consonant intervals.
7. Experiments in which are produced from the above (sec. 6) curves the Motions of a Molecule of Air when it is animated with the resultant action of the six elementary vibrations forming a musical note; or is set in motion by the combined action of sonorous vibrations forming various consonant intervals.

### 1. *An Experimental Confirmation of Fourier's Theorem as applied to the decomposition of the vibrations of a composite sonorous wave into its elementary pendulum-vibrations.*

A simple sound is a sound which has only one pitch. Such a sound is produced when, with a bow, we gently vibrate the prongs of a tuning-fork and bring them near a cavity which resounds to the fork's fundamental tone. An almost pure simple sound can be obtained by softly blowing a closed organ-pipe. On examining the nature of the vibratory motions of the

* This paper is the fifth, in the series of those on Acoustics, already published in this Journal. The preceding papers, however, were not numbered.

Sections 1, 2, 3, 5, 6 and 7 of this paper were read before the National Academy of Sciences during the session of November, 1873. Section 4 was read before the Academy on April 21, 1874.

prongs of the fork* and of the molecules of air in the resounding cavity† and in the closed organ-pipe,‡ we find that each of these vibrations follows the same law of reciprocating motion as governs the vibrations of a freely-swinging pendulum. But other bodies, for instance, the free-reeds of organ-pipes and of melodeons,§ vibrate like the pendulum, yet we can decompose the vibrations they produce in the air into many separate pendulum-vibrations, each of which produces in the ear a simple sound of a definite pitch; thus, we see that a pendulum-vibrating body, when placed in certain relations to the air on which it acts, may give rise to highly composite sounds. It is, therefore, evident that we cannot always decide as to the simple or composite character of a vibration reaching the ear solely from the determination of the motion of the body originating the sound, but we are obliged to investigate the character of the molecular motions of the air near the ear, or of the motion of a point on the drum of the ear itself, in order to draw conclusions as to the simple or composite character of the sensation which may be produced by any given vibratory motion. Although we cannot often detect in the ascertained form of an aerial vibration all the elementary pendulum-vibrations, and thus predetermine the composite sensation connected with it; yet, if we find that the aerial vibration is that of a simple pendulum, we may surely decide that we will receive from it only

* In my course of lectures on Acoustics, I thus show to my students that the prong of a tuning-fork vibrates like a pendulum. I take two of Lissajous' reflecting forks, giving, say, the major third interval, and with them I obtain on a screen the curve of this interval in electric light. On a glass plate I have photographed the above curve of the major third passing through a set of rectangular coördinates formed of the sines of two circles whose circumferences are respectively divided into 20 and 25 equal parts. I now place this plate over the condensing-lens of a vertical lantern and obtain on the screen the curve, the circles and their net of coördinates. Suspended over the lantern is a Blackburn's compound pendulum, which is so constructed that its "bob" cannot rotate around its axis. The bob is hollow and a curved pipe leads from its bottom to one side of the pendulum. The pendulum is now deflected into a plane at 45° with its two rectangular planes of vibration so that the end of the curved pipe coincides with the beginning of the curve over the lantern. The bob of the pendulum is fastened with a fine cord in this position and fine hour-glass sand is poured into it; the cord is now burned and the sand is delivered from the pipe, as the swinging pendulum gives the resultant of its motions in the two planes of vibration, while the photographed curve on the lantern is progressively covered with the sand if the tines of the two vibrations of the pendulum are to each other as 4 to 5.

† Helmholtz, Tonempfindungen, 1857, p. 75. Crelle's Jour. für Math. Bd. lvii.

‡ See Mach's Optisch-Akustische Versuche, Prag., 1873, p. 91. Die Stroboskopische Darstellung der Luftschwingungen.

§ The Rev. S. B. Dod, one of the trustees of the Stevens Institute, has recently made an experiment which neatly shows this. He silvered the tips of two melodeon reeds and then vibrated them in planes at right angles to each other, while a beam of light was reflected from them. The resultant figure of their vibrations is the same as that obtained by two Lissajous' forks placed in the same circumstances and having the same musical interval between them as that existing between the reeds.

the sensation of a simple sound. Thus, if we arm the prong of a tuning-fork with a point, and draw this point on a lamp-blackened surface with a uniform motion, and in a direction parallel to the axis of the fork, we shall obtain on the surface a sinusoidal or harmonic curve;* and this curve can only be produced by the prongs of the fork vibrating with the same kind of motion as that of a freely-swinging pendulum. If we now bring this vibrating fork near the mouth of a glass vessel whose mass of air responds to the tone of the fork and, by the method of Mach, examine the vibratory-motions of the air, we shall see it swinging backward and forward; and by combining these vibrations with the rectangular vibrations of forks placed outside of the vessel we shall obtain the curves of Lissajous. If the membrane of the drum of the ear be placed in connection with the resounding cavity, it must necessarily partake of the motion of the air which touches it, and ultimately the auditory nerve fibrillæ are shaken in the same manner, and we receive the sensation† of a simple sound. Here the mind naturally inquires the reason of this connection existing between the sensation of a simple sound and the pendulum-vibration. It has always appeared to me that the explanation of this invariable connection is that the pendulum-vibration is the simplest vibratory motion that the molecules of elastic matter can partake of, and that the connection of the sensation with the mode of vibration is the connection between the simplest sensation perceived through the intervention of the trembling nerves, and the simplest vibration which they can experience. Indeed, the pendulum-vibration is the only one which produces the sensation of sound, for if any other recurring vibration enters the ear it is decomposed by the ear into its elementary pendulum-vibrations, and if it cannot be so decomposed then the given vibration is not recurring and does not produce in us the sensation of sound, but causes that which we denominate as noise. This remarkable connection between a simple sound and the pendulum, or harmonic, vibration, together with the fact of the power of the ear to decompose the motions of a composite sonorous wave into its vibratory elements, was thus distinctly enunciated by Ohm. *The ear has the sensation of a simple sound only when it receives a pendulum-vibration, and it decomposes any other periodic motion of the air into a series of pendulum-vibrations, each of which corresponds to the sensation of a simple sound.*

* The equation of this curve is $y = a \sin\left(\frac{2\pi x}{\lambda} + \alpha\right)$. The length on the axis of one recurring period of the curve is $\lambda$; the constant $a$ is the maximum ordinate, or amplitude. The form of the curve is not affected by $\alpha$, but any change in its value slides the whole curve along the axis of $x$. It is interesting to observe that this curve expresses the annual variation of temperature in the temperate zones.

† See Helmholtz on the distinction between a sensation and a perception. Tonempfindungen, p. 101.

We have seen that the harmonic curve is the curve which corresponds to the motion which causes the sensation of a simple sound, but a molecule of vibrating air or a point on the tympanic membrane may be actuated by vibratory motions, which, when projected on a surface moving near them, will develop curves which depart greatly from the simplicity of the harmonic, or curve of sines;* but nevertheless these curves will alway be periodic if the sensation corresponding to their generating motions is that of sound. Now Fourier has shown, and states in his theorem, that any periodic curve can always be reproduced by corresponding harmonic curves (often infinite in number) having the same axis as the given curve and having the lengths of their recurring periods as 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., of the given curve; and the only limitation to its irregularity is that its ordinates must be finite, and that the projection on the axis, of a point moving in the curve, must always progress in the same direction. Fourier demonstrates that the given curve can only be reproduced by one special combination, and shows that by means of definite integrals one can assign the definite sinusoids with their amplitudes and differences of phase. Now Helmholtz† has shown that differences of phase in the constituent elementary sounds do not alter the character of the composite sound, and therefore, that although the forms of the curve corresponding to one and the same composite sound may be infinite in variety (by reason of differences in phase and amplitude in the component curves), yet the composite sound is always resolved into the same elements. This experimental result of Helmholtz also conforms to the theorem of Fourier in reference to the curves projected by such motions; for he has shown that only one series of sinusoidal resolution is possible.

Fourier's theorem can be expressed as follows: The constants C, $C_1$, $C_2$, &c., and $a_1$, $a_2$, &c., can be determined so that a period of the curve can be defined by the following equation:‡

$$y = C + C_1 \sin\left(\frac{2\pi x}{\lambda} + a_1\right) + C_2 \sin\left(2\frac{2\pi x}{\lambda} + a_2\right) + \cdots$$

But Fourier's theorem is the statement of a mathematical possibility, and it does not necessarily follow that it can be immediately translated into the language of dynamics without experimental confirmation, for, as Helmholtz remarks, "That mode of decomposition of vibratory forms, such as the theorem of Fourier describes and renders possible—is it only a

* In section 5 of this paper, I have constructed several important curves corresponding to composite vibrations.

† Tonempfindungen, p. 190 *et seq.*

‡ For other and more convenient forms of expression of this theorem, as well as for a demonstration of it, see pp. 52 and 60 of Donkin's Acoustics—the most admirable work ever written on the mathematical theory of sound.

mathematical fiction, admirable because it renders computation facile, but not corresponding necessarily to anything in reality? Why consider the pendulum-vibration as the irreducible element of all vibratory motion? We can imagine a whole, divided in a multitude of different ways; in a calculation we may find it convenient to replace the number 12 by 8+4, in order to bring 8 into view; but it does not necessarily follow that 12 should always and necessarily be considered as the sum of 8+4. In other cases it may be more advantageous to consider the number as the sum of 7+5."

The mathematical possibility, established by Fourier, of decomposing any sonorous motion into simple vibrations, cannot authorize us to conclude that this is the only admissible mode of decomposition, if we cannot prove that it has a signification essentially real. The fact, that the ear effects that decomposition, induces one, nevertheless, to believe that this analysis has a signification, independent of all hypothesis, in the exterior world. This opinion is also confirmed precisely by the fact stated above, that this mode of decomposition is more advantageous than any other in mathematical researches. For the methods of demonstration which comport with the intimate nature of things, are naturally those which lead to theoretic results the most convenient and the most clear."

The theorem of Fourier translated into the language of dynamics would read as follows: "*Every periodic vibratory motion can always, and always in one manner, be regarded as the sum of a certain number of pendulum-vibrations.*"

Now we have seen that any periodic vibratory motion, which has the proper velocity, will cause the sensation of a musical note, and that a pendulum-vibration gives the sensation of a simple sound;* therefore, if Fourier's theorem is applicable to the composition and decomposition of a composite sonorous wave, it will be thus related to the phenomena of sound: "*Every vibratory motion in the auditory canal, corresponding to a musical sound, can always, and always in one manner, be considered as the sum of a certain number of pendulum-vibrations, corresponding to the elementary sounds of the given musical note.*"

Heretofore we have called in the aid of the sensations,—assumed to be received through the motions of the co-vibrating

* Professor Donkin, in his Acoustics, Oxford, 1870, p. 11, advises the use of *tone* to designate a simple sound, and the word *note* to distinguish a composite sound. His reasons are "that *tone* (Gr. τόνος) really means *tension*, and the effect of tension is to determine the *pitch* of the sound of a string;" while a musical *note* is generally a composite sound. Professor Donkin further states: "Helmholtz uses the words *klang* and *ton* to signify compound and simple musical sounds. We have followed him in adopting the latter term. But such a sound as that of the human voice could hardly in English be called a *clang*, without doing too much violence to established usage."

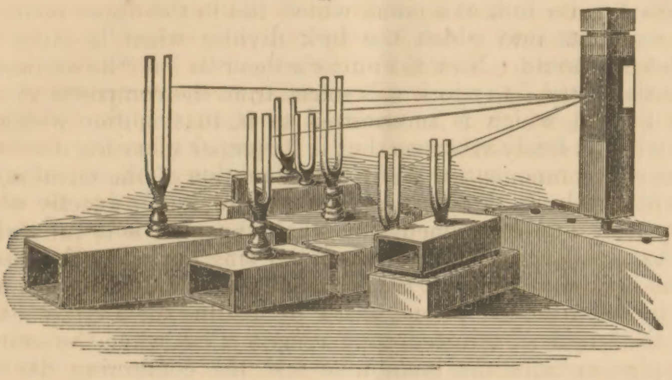
parts of the ear,—to help us in our determination of the simple or composite character of a given vibratory motion; but Fourier's theorem does not refer to the subjective effects on the organ of hearing,—the dynamic function of whose parts are yet very imperfectly understood. Ohm's theorem, on the other hand, refers entirely to these subjective phenomena of the ear's analysis of a complex sensation into its simple elements. As Fourier's theorem refers only to the decomposition of a composite recurring vibration into its elementary pendulum-vibrations, it has nothing to do with the physiological fact of the co-relation of the pendulum-vibration and the simplest auditory sensation; though this well ascertained relation gives us the privilege of using this sensation as an indicator of the existence of an aerial pendulum-vibration. Hence, as Fourier's theorem is entirely independent of our sensations, we must endeavor to verify it directly by experiments, which must perform *the actual* decomposition of the composite periodic motion of *a point* into its elementary pendulum-vibrations. But many difficulties present themselves when we would bring to the test of experiment the dynamic signification of Fourier's theorem. For example, the composite sound-vibration, on which we would experiment, emanates from a multitude of vibrating points; parts of the resultant wave surface differ in their amplitudes of vibration; while points equally removed from one and the same point of the body originating the vibrations, may differ in their phases of vibration; so that when such a wave falls upon co-vibrating bodies which present any surface, the effects produced are the results of extremely complex motions. The mind sees at once the difference between this complicated conception and the simple one embodied in the statement of the dynamic application of Fourier's theorem.

As the mathematician decomposes *seriatim* every point of the recurring curve into its harmonic elements, so the physicist, in confirming the dynamic application of Fourier's theorem, should decompose into its simple pendulum-vibrations the composite vibratory motion which such a curve represents, and indeed *reproduces* when it is drawn with a uniform motion under a slit in a diaphragm which exposes to view only a point of the curve at once. Therefore, only one vibrating point of the composite sonorous wave should be experimented on, and the composite vibratory motion of this point should be conveyed along lines to points of elastic bodies which can only partake of simple pendulum-vibrations. All of these essential conditions I have succeeding in securing in the following arrangement of apparatus.

A loose inelastic membrane—(thin morocco-leather does well)—was mounted in a frame and placed near a reed-pipe; or, as in

other experiments, the membrane was placed over an opening in the front of the wooden chamber of a Grenié's free-reed pipe. The ends of several fine fibers from a silk-worm's cocoon were brought neatly together and cemented to one and the same point of the membrane, while the other ends of these fibers were attached to tuning-forks mounted on their resonant boxes, as shown in fig 1. In the experiment which I shall now

1.



describe, eight forks were thus connected with one point of the membrane. The fundamental tone of the pipe was $Ut_2$, of 128 vibrations per second; and the pipe was brought into accurate unison with a fork giving this sound.* The forks connected with the membrane were the harmonic series of $Ut_2$, $Ut_3$, $Sol_3$, $Ut_4$, $Mi_4$, $Sol_4$, $B^{b-}_4$, $Ut_5$. In the first stage of the experiment we will suppose that the fibers are but slightly stretched; then, on sounding the pipe, all the fibers at once break up into exquisite combinations of ventral segments. If the sunshine fall upon a vibrating fiber and we look on it obliquely, in the direction of its length, we shall see ventral segments superimposed on ventral segment in beautiful and changing combinations. On gradually tightening the fibers, we diminish the number of their nodes, and on reaching a certain degree of tension with fibers 1 m. long, I have seen them all vibrating with single ventral segments. On increasing the tension, the amplitudes of these single segments gradually diminish and at last disappear entirely, so far as the unaided eye can discern, and then we have reached the conditions required in our experimental confirmation.

* Since the number of beats per second given by any harmonic (of a pipe out of tune with its harmonic series of forks) will be as the order of the harmonic, it is better to tune a reed to unison with a fork giving one of its higher harmonics. I generally used the $Sol_3$ fork, or the 3d harmonic.

The point of the membrane to which the fibers are attached is actuated by a motion which is the resultant of all of the elementary pendulum-vibrations existing in the composite sonorous wave, and the composite vibrations of this point are sent through each of the fibers to its respective fork. Thus, each fiber transmits to its fork the same composite vibratory motion, while each fork can only vibrate so as to give the simple pendulum-vibration of a simple sound, for each fiber is attached to its fork at a point which lies in the upper node of the segments into which the fork divides when it gives its higher harmonic. Now if Fourier's theorem has "an existence essentially real," any fork will select from the composite vibratory motion, which is transmitted to it, that motion which it has when it freely vibrates; but if its proper vibration does not exist as a component of the resultant motion of the membrane, it will not be in the least affected. Now this is exactly what happens in our experiment, for when the pipe is in tune with the harmonic series of forks, the latter sing out when the membrane is vibrated; but if the forks be even slightly thrown out of tune with the membrane, either by loading them, or by altering the length of the reed, they remain silent when the sounding pipe agitates the membrane and the connecting fibers.* Thus have I shown that the dynamic application of Fourier's theorem has "an existence essentially real."

It is indeed very interesting and instructive thus to observe in one experiment the analysis and synthesis of a composite sound. On sounding the reed it sets in vibration all the forks of the harmonic series of its fundamental note, and after the reed has ceased to sound, the forks continue to vibrate and their elementary simple sounds blend into a note which approximately reproduces the timbre of the reed-pipe. If we could by any means obtain all of the elementary vibrations, and have them with their relative intensities correctly preserved, we should have an echo of the sound of the reed after the latter had ceased to vibrate; but the impossibility of thus obtaining the highest components of the reed, and the difficulty of reproducing the relative intensities of the harmonics in the co-vibrating forks, allow us but partially to accomplish this effect.

### 2. *An Experimental Illustration of Helmholtz's Hypothesis of Audition.*

The experiment, which we have just described, beautifully illustrates the hypothesis of audition framed by Helmholtz to account for this—among other facts,—that the ear can decompose a composite sound into its sonorous elements. Helmholtz

* See section 4 of this paper for an account of the degree of precision of this method of sonorous analysis.

founds his hypothesis on the supposition that the rods of Corti, in the ductus cochlearis, are bodies which co-vibrate to simple sounds: somewhat, I imagine, as loaded strings* of graded lengths and diameters would act in similar circumstances. The vibrations of the composite wave fall upon the membrane placed near the reed as they fall upon the membrane of the tympanum; and these vibrations are sent through the stretched fibers, (or delicate splints of rye-straw, which I have sometimes used,) from the membrane to the tuned forks, as they are sent from the membrana tympani through the ossicles and fluids of the ear to the rods of Corti. The composite vibration is decomposed into its vibratory elements by the co-vibration of those forks whose vibratory periods exist as elements of the composite wave motion; so the composite sound is decomposed into its sonorous elements by the co-vibrations of the rods of Corti, which are tuned to the elementary sounds which exist in the composite sonorous vibration. The analogy can be carried yet further by placing the forks in line and in order of ascending pitch, and attaching to each fork a sharply-pointed steel filament. If the arm be now stretched near the forks, so that the points of the filaments nearly touch it at points along its length, then any fork will indicate its co-vibration by the fact of its pricking the skin of the arm, and the localization of this pricking will tell us which of the series of forks entered into vibration. The rods of Corti shake the nerve filaments attached to them, and thus specialize the position in the musical scale of the elements of a composite sonorous vibration. Thus a complete analogy is brought into view between our experiment and Helmholtz's comprehensive hypothesis of the mode of audition.

3. *Experiments on the supposed Auditory Apparatus of the Culex Mosquito.*

Ohm states in his proposition that the ear experiences a simple sound only when it receives a pendulum-vibration, and that it decomposes any other periodic motion of the air into a series of pendulum-vibrations, to each of which corresponds the sensation of a simple sound. Helmholtz, fully persuaded of the truth of this proposition, and seeing its intimate connection with the theorem of Fourier, reasoned that there must be a cause for it in the very dynamic constitution of the ear; and the previous discovery by the Marquis of Corti of several thousand† rods of graded sizes in the *ductus cochlearis* indicated

* For discussions of the vibratory phenomena of loaded strings, see Donkin's Acoustics, p. 139; and Helmholtz's Tonempfindungen, p. 267.

† "But all of the propositions on which we have based the theory of consonance and dissonance rests solely on a minute analysis of the sensations of the ear. This analysis could have been made by any cultivated ear, without the aid of

to Helmholtz that these were suitable bodies to effect the decomposition of a composite sonorous wave by their co-vibrating with its simple harmonic elements. This supposed function of the Corti organ gave a rational explanation of the theorem of Ohm, and furnished "a leading thread" which conducted Helmholtz to the discoveries contained in his renowned work, "Die Lehre von den Tonempfindungen."* In this book he first gave the true explanation of timbre, and revealed the hidden cause of musical harmony, which, since the days of Pythagoras, had remained a mystery to musicians and a problem to philosophers.

It may, perhaps, never be possible to bring Helmholtz's hypothesis of the mode of audition in the higher vertebrates to the test of direct observation, from the apparent hopelessness of ever being able to experiment on the functions of the parts of the inner ear of mammalia. The cochlea, tunneled in the hard temporal bone, is necessarily difficult to dissect, and even when a view is obtained of the organ of Corti, its parts are rarely *in situ;* and, moreover, they have already had their natural structure altered by the acid with which the bone has been saturated to render it soft enough for dissection and for the cutting of sections for the microscope.

As we descend in the scale of development, from the higher vertebrates, we observe the parts of the outer and middle ear disappearing, while at the same time we see the inner ear gradually advancing toward the surface of the head. The external ear, the auditory canal, the tympanic membrane, and with the latter the now useless ossicles, have disappeared in the lower vertebrates, and there remains but a rudimentary labyrinth.

Although the homological connections existing between the vertebrates and articulates, even when advocated by naturalists, are certainly admitted to be imperfect, yet we can hardly suppose that the organs of hearing in the articulates will remain stationary or retrograde, but rather that the essential parts of their apparatus of audition, and especially that part which receives the aerial vibrations, will be more exposed than in higher organisms. Indeed, the very minuteness of the greater part of the articulates would indicate this, for a tympanic membrane placed in vibratory communication with a modified

theory, but the leading-thread of theory, and the employment of appropriate means of observation, have facilitated it in an extraordinary degree.

"Above all things I beg the reader to remark that the hypothesis on the co-vibration of the organs of Corti has no immediate relation with the explanation of consonance and dissonance, which rests solely on the facts of observation, on the beats of harmonics and of resultant sounds."—Helmholtz, Tonempfindungen, p. 342.

* According to Waldeyer, there are 6,500 inner and 4,500 outer pillars in the organ of Corti.

labyrinth, or even an auditory capsule with an outer flexible covering, would be useless to the greater number of insects for several reasons; first, such an apparatus, unless occupying a large proportion of the volume of an insect, would not present surface enough for this kind of receptor of vibrations: and secondly, the minuteness of such a membrane would render it impossible to co-vibrate with those sounds which generally occur in nature, and which the insects themselves can produce; similarly, all non-aquatic vertebrates have an inner ear formed so as to bring the aerial vibrations, which strike the tympanic membrane, to bear with the greatest effect on the auditory nerve filaments,* and the minuteness of insects also precludes this condition. Finally, the hard test, characteristic of the articulates, sets aside the idea that they receive the aerial vibrations through the covering of their bodies, like fishes, whose bodies are generally not only larger and far more yielding, but are also immersed in water which transmits vibrations with $4\frac{1}{4}$ times the velocity of the same pulses in air and with a yet greater increase in intensity. For these reasons, I imagine that those articulates which are sensitive to sound, and also emit characteristic sounds, will prove to possess receptors of vibrations external to the general surface of their bodies, and that the proportions and situation of these organs will comport with the physical conditions necessary for them to receive and transmit vibrations to the interior ganglia.

Naturalists, in their surmises as to the positions and forms of the organ of hearing in insects, have rarely kept in view the important consideration of those physical relations which the organ must bear to the aerial vibrations producing sound, and which we have already pointed out. The mere descriptive anatomist of former years could be satisfied with his artistic faculty for the perception of form, but the student of these days can only make progress by constantly studying the close relations which necessarily exist between the minute structure of the organs of an animal and the forces which are acting in the animal, and which traverse the medium in which the animal lives. The want of appreciation of these relations, together with the fact that many naturalists are more desirous to describe many new forms than to ascertain the function of one well known form, which may exist in all animals of a class, has tended to keep many departments of natural history in the condition of mere descriptive science. Those who are not professed naturalists appreciate this perhaps more than the naturalists themselves, who are imbued with that enthusiasm which always comes with the earnest study of any one department of nature; for the perusal of those long and laboriously precise descriptions of forms of organs without the slightest

* See section 4 of this paper.

attempt, or even suggestion, as to their uses, affects a physicist with feelings analogous to those experienced by one who peruses a well classified catalogue descriptive of physical instruments, while of the uses of these instruments he is utterly ignorant.

The following views, taken from the "Anatomy of the Invertebrata by C. Th. v Siebold," will show how various are the opinions of naturalists as to the location and form of the organs of hearing in the *Insecta*. "There is the same uncertainty concerning the organs of audition (as concerning the olfactory organs). Experience having long shown that most insects perceive sounds, this sense has been located sometimes in this and sometimes in that organ. But in their opinion, it often seems to have been forgotten, or unthought of, that there can be no auditory organ without a special auditory nerve, which connects directly with an acoustic apparatus capable of receiving, conducting and concentrating the sonorous undulations. (The author who has erred most widely in this respect is L. W. Clarke, in Mag. Nat. Hist., Sept, 1838, who has described at the base of the antennæ of *Carabus nemoralis* Illig. an auditive apparatus composed of an *Auricula*, a *Meatus auditorius externus* and *internus*, a *Tympanum* and *Labyrinthus*, of all of which there is not the least trace. The two white convex spots at the base of the antennæ of *Blatta orientalis*, and which Treviranus has described as auditory organs, are, as Burmeister has correctly stated, only rudimentary accessory eyes. Newport and Goureau think that the antennæ serve both as tactile and as auditory organs. But this view is inadmissible, as Erichson has already stated, except in the sense that the antennæ, like all solid bodies, may conduct sonorous vibrations of the air; but, even admitting this view, where is the auditory nerve? for it is not at all supposable that the antennal nerve can serve at the same time the function of two distinct senses.)

"Certain Orthoptera are the only Insecta with which there has been discovered, in these later times, a single organ having the conditions essential to an auditory apparatus. This organ consists, with the Acrididæ, of two fossæ or conchs, surrounded by a projecting horny ring, and at the base of which is attached a membrane resembling a tympanum. On the internal surface of this membrane are two horny processes, to which is attached an extremely delicate vesicle filled with a transparent fluid and representing a membranous labyrinth. This vesicle is in connection with an auditory nerve which arises from the third thoracic ganglion, forms a ganglion on the tympanum, and terminates in the immediate neighborhood of the labyrinth by a collection of cuneiform. staff-like bodies with very finely-pointed extremities (primitive nerve-fibers?), which are sur-

rounded by loosely-aggregated, ganglionic globules. (This organ has been taken for a soniferous apparatus by Latreille. J. Müller was the first who fortunately conceived that with *Gryllus hieroglyphus* this was an auditory organ. He gave, however, the interpretation only as hypothetical; but I have placed it beyond all doubt by careful researches made on *Gomphoceros*, *Oedipoda*, *Podisma*, *Caloptenus* and *Truxalis*.")

"The Locustidæ and Achetidæ have a similar organ, situated in the fore-legs directly below the coxo-tibial articulation. With a part of the Locustidæ (*Meconema*, *Barbitistes*, *Phaneroptera*, *Phylloptera*), there is on each side of this point a fossa, while with another portion of this family there are, at this same place, two more or less spacious cavities (auditory capsules) provided with orifices opening forward. These fossæ and these cavities have each, on their internal surface, a long-oval tympanum. The principal trachean trunk of the leg passes between two tympanums, and dilates, at this point, into a vesicle whose upper extremity is in connection with a ganglion of the auditory nerve. This last arises from the first thoracic ganglion, and accompanies the principal nerve of the leg. From this ganglion in question passes off a band of nervous substance, which stretches along the slightly excavated anterior side of the trachean vesicle. Upon this band is situated a row of transparent vesicles containing the same kind of cuneiform, staff-like bodies, mentioned as occurring with the Acrididæ. The two large trachean trunks of the fore-legs open by two wide, infundibuliform orifices on the posterior border of the prothorax, so that here, as with the Acrididæ, a part of this trachean apparatus may be compared to a *Tuba Eustachii*. With the Achetidæ, there is, on the external side of the tibia of the fore-legs, an orifice closed by a white, silvery membrane (tympanum), behind which is an auditory organ like that just described. (With *Acheta achatina* and *italica*, there is a tympanum of the same size, on the internal surface of the legs in question; but it is scarcely observable with *Acheta sylvestris*, *A. domestica* and *A. campestris*.)"

Other naturalists have placed the auditory apparatus of diurnal lepidoptera in their club-shaped antennæ; of bees at the root of their maxillæ; of *Melolontha* in their antennal plates; of *Locusta viridissima* in the membranes which unite the antenna with the head.

I think that Siebold assumes too much when he states that the existence of a tympanic membrane is the only test of the existence of an auditory apparatus. It is true that such a test would apply to the non-aquatic vertebrates, but their homologies do not extend to the articulates; and besides, any physicist can not only conceive of, but can actually construct other

receptors of aerial vibrations, as I will soon show by conclusive experiments. Neither can I agree with him in supposing that the antennæ are only tactile organs, for very often their position and limited motion would exclude them from this function;* and, moreover, it has never been proved that the antennæ, which differ so much in their forms in different insects, are always tactile organs. They may be used as such in some insects; in others, they may be organs of audition; while in other insects they may, as Newport and Goureau surmise, have both functions; for, even granting that Müller's law of the specific energy of the senses extends to the insects, yet the anatomy of their nervous system is not sufficiently known to prevent the supposition that there may be two distinct sets of nerve fibers in the antennæ or in connection with their bases; so that the antennæ may serve both as tactile and as auditory organs: just as the hand, which receives at the same time the impression of the character of the surface of a body and of its temperature; or, like the tongue, which at the same time distinguishes the surface, the form, the temperature and the taste of a body. Finally, I take objection to this statement: "Newport and Goureau think that the antennæ serve both as tactile and auditory organs. But this view is inadmissible, as Erichson has already stated, except in the sense that the antennæ, like all solid bodies, may conduct sonorous vibrations of the air." Here, evidently, Siebold had not in his mind the physical relations which exist between two bodies which give exactly the same number of vibrations; for it is well known that when one of them vibrates, the other will be set into vibration by the impacts sent to it through the intervening air. Thus, if the fibrillæ on the antennæ of an insect should be tuned to the different notes of the sound emitted by the same insect, then when these sounds fell upon the antennal fibrils, the latter would enter into vibration with those notes of the sound to which they were severally tuned; and so it is evident that not only could a properly constructed antenna serve as a receptor of sound, but it would also have a function not possible in a membrane; that is, it would have the power of analyzing a composite sound by the co-vibration of its various fibrillæ to the elementary tones of the sound.

The fact that the existence of such an antenna is not only supposable but even highly probable, taken in connection with an observation I have often made in looking over entomological collections; viz: that fibrillæ on the antennæ of noctural insects are highly developed, while on the antennæ of diurnal insects they are either entirely absent or reduced to mere rudimentary filaments, caused me to entertain the hope that I should

* Indeed, they are often highly developed in themselves while accompanied by *palpi*, which are properly placed, adequately organized and endowed with a range of motion suitable to an organ intended for purposes of touch.

be able to confirm my surmises by actual experiments on the effects of sonorous vibrations on the antennal fibrillæ; also, the well known observations of Hensen encouraged me to seek in aerial insects for phenomena similar to those he had found in the decapod, the *Mysis*, and thus to discover in nature an apparatus whose functions are the counterpart of those of the apparatus with which I gave the experimental confirmation of Fourier's theorem, and similar to the supposed functions of the rods of the organ of Corti.

The beautiful structure of the plumose antennæ of the male *Culex Musquito* is well known to all microscopists; and these organs at once recurred to me as suitable objects on which to begin my experiments. The antennæ of these insects are twelve-jointed and from each joint radiates a whorl of fibrils, and the latter gradually decrease in their lengths as we proceed from those of the second joint from the base of the antenna to those of the second joint from the tip. These fibrils are highly elastic and so slender that their lengths are over three hundred times their diameters. They taper slightly, so that their diameter at the base is to the diameter near the tip as 3 to 2.

I cemented a live male mosquito with shellac to a glass slide and brought to bear on various fibrils a $\frac{1}{4}$th objective. I then sounded successively, near the stage of the microscope, a series of tuning-forks with the openings of their resonant boxes turned toward the fibrils. On my first trials with an $Ut_4$ fork, of 512 v. per sec., I was delighted with the results of the experiments, for I saw certain of the fibrils enter into vigorous vibration, while others remained comparatively at rest.

The table of experiments which I have given is characteristic of all of the many series which I have made. In the first column (A) I have given the notes of the forks in the French notation, which König stamps upon his forks. In the second (B) are the amplitudes of the vibrations of the end of the fibril in divisions of the micrometer scale; and in column (C) are the values of these divisions in fractions of a millimeter.

| A. | B. | C. |
|---|---|---|
| $Ut_2$ | ·5 div. | ·0042 mm. |
| $Ut_3$ | 2·5 | ·0200 |
| $Mi_3$ | 1·75 | ·0147 |
| $Sol_3$ | 2·0 | ·0168 |
| $Ut_4$ | 6·0 | ·0504 |
| $Mi_4$ | 1·5 | ·0126 |
| $Sol_4$ | 1·5 | ·0126 |
| $B^{\flat}_4$– | 1·5 | ·0126 |
| $Ut_5$ | 2·0 | ·0168 |

The superior effect of the vibrations of the $Ut_4$ fork on the fibril is marked, but thinking that the differences in the ob-

served amplitudes of the vibrations might be owing to differences in the intensities of the various sounds, I repeated the experiment, but vibrated the forks which gave the greater amplitudes of co-vibration with the lowest intensities; and although I observed an approach toward equality of amplitude, yet the fibre gave the maximum swings when $Ut_4$ was sounded, and I was persuaded that this special fibril was tuned to unison with $Ut_4$ or to some other note within a semitone of it. The differences of amplitude given by $Ut_4$ and $Sol_3$ and $Mi_4$ are considerable, and the table also brings out the interesting observation that the lower ($Ut_3$) and the higher ($Ut_5$) harmonics of $Ut_4$ cause greater amplitudes of vibration than any intermediate notes. As long as a universal method for the determination of the relative intensities of sounds of different pitch remains undiscovered, so long will the science of acoustics remain in its present vague qualitative condition.* Now, not having the means of equalizing the intensities of the vibrations issuing from the various resonant boxes, I adopted the plan of sounding, with a bow, each fork with the greatest intensity I could obtain. I think that it is to be regretted that König did not adhere to the form of fork, with *inclined prongs*, as formerly made by Marloye; for with such forks one can always reproduce the same initial intensity of vibration by separating the prongs by means of the same cylindrical rod which is drawn between them. Experiments similar to those already given revealed a fibril tuned to such perfect unison with $Ut_3$ that it vibrated through 18 divisions of the micrometer or ·15 mm., while its amplitude of vibration was only 3 div. when $Ut_4$ was sounded. Other fibrils responded to other notes, so that I infer from my experiments on about a dozen mosquitos that their fibrils are tuned to sounds extending through the middle and next higher octave of the piano.

* I have recently made some experiments in this direction, which show the possibility of eventually being able to express the intensity of an aerial vibration directly in fraction of Joule's Dynamical Unit, by measuring the heat developed in a slip of sheet rubber stretched between the prongs of a fork and enclosed in a compound thermo-battery. The relative intensities of the aerial vibration produced by the fork when engaged in heating the rubber and when the rubber is removed, can be measured by the method I described in the Amer. Jour. Sci., Feb., 1873. Of course if we can determine the amount of heat produced per second by a known fraction of the intensity, we have the amount produced by the vibration with its entire intensity. Then means can be devised by which the aerial vibration produced by this fork can always be reproduced with the same intensity. This intensity, expressed in fraction of Joule's unit, is stamped upon the apparatus, which ever afterward serves as a true measure for obtaining the intensities of the vibrations of all simple sounds having the same pitch as itself. The same operation can be performed on other forks of different pitch, and so a series of intensities of different periods of vibration is obtained expressed in a corresponding series of fractions of Joule's unit. Recent experiments have given 1/[illegible]th of a Joule's unit as the approximate dynamic equivalent of ten seconds of aerial vibrations produced by an $Ut_4$ fork, set in motion by intermittent electro-magnetic action and placed before a resonator.

To subject to a severe test the supposition I now entertained, that the fibrils were tuned to various periods of vibration, I measured with great care the lengths and diameters of two fibrils, one of which vibrated strongly to $Ut_3$, the other as powerfully to $Ut_4$; and from these measures I constructed in homogeneous pine wood two gigantic models of the fibrils; the one corresponding to the $Ut_3$ fibril being about one meter long. After a little practice I succeeded in counting readily the number of vibrations they gave when they were clamped at one end and drawn from a horizontal position. On obtaining the ratio of these numbers, I found that it coincided with the ratio existing between the numbers of vibrations of the forks to which co-vibrated the fibrils of which these pine rods were models.

The consideration of the relations which these slender, tapering, and pointed fibrils must have to the aerial pulses acting on them, led me to discoveries in the physiology of audition which I imagine are entirely new. If a sonorous wave falls upon one of these fibrils so that its wave-front is at right angles to the fibril, and hence the direction of the pulses in the wave are in the direction of the fibril's length, the latter cannot be set in vibration; but if the vibrations in the wave are brought more and more to bear athwart the fibril it will vibrate with amplitudes increasing until it reaches its maximum swing of co-vibration, when the wave-front is parallel to its length and therefore the direction of the impulses on the wave are at right angles to the fibril. These curious surmises I have confirmed by many experiments made in the following manner. A fork which causes a strong co-vibration in a certain fibril is brought near the microscope, so that the axis of the resonant box is perpendicular to the fibril and its opening is toward the microscope. The fibril, in these circumstances, enters into vigorous vibration on sounding the fork; but, on moving the box around the stage of the microscope so that the axis of the box always points toward the fibril, the amplitudes of vibration of the fibril gradually diminish, and when the axis of the box coincides with the length of the fibril, and therefore the sonorous pulses act on the fibril in the direction of its length, the fibril is absolutely stationary and even remains so when the fork, in this position, is brought quite close to the microscope. These observations at once revealed to me a new function of these organs; for if, for the moment, we assume that the antennæ are really the organs which receive aerial vibrations and transmit them to an auditory capsule, or rudimentary labyrinth, then these insects must have the faculty of the perception of the direction sound more highly developed than in any other class of animals. The following experiments will show the force of

this statement and at the same time illustrate the manner in which these insects determine the direction of a sonorous center. I placed under the microscope a live mosquito, and kept my attention fixed upon a fibril which co-vibrated to the sound of a tuning-fork, which an assistant placed in unknown positions around the microscope. I then rotated the stage of the instrument until the fibril ceased to vibrate, and then drew a line on a piece of paper, under the microscope, in the direction of the fibril. On extending this line, I found that it always cut within 5° of the position of the source of the sound. The antennæ of the male mosquito have a range of motion in a horizontal direction, so that the angle included between them can vary considerably inside and outside of 40°,* and I conceive that this is the manner in which these insects during night direct their flight toward the female. The song of the female vibrates the fibrillæ of one of the antennæ more forcibly than those of the other. The insect spreads the angle between his antennæ, and thus, as I have observed, brings the fibrillæ, situate within the angle formed by the antennæ, in a direction approximately parallel to the axis of the body. The mosquito now turns his body in the direction of that antenna whose fibrils are most affected, and thus gives greater intensity to the vibrations of the fibrils of the other antenna. When he has thus brought the vibrations of the antennæ to equality of intensity, he has placed his body in the direction of the radiation of the sound, and he directs his flight accordingly; and from my experiments it would appear that he can thus guide himself to within 5° of the direction of the female.

Some may assume from the fact of the co-vibration of these fibrils to sounds of different pitch, that the mosquito has the power of decomposing the sensation of a composite sound into its simple components, as is done by the higher vertebrates; but I do not hold this view, but believe that the range of co-vibration of the fibrils of the mosquito is to enable it to apprehend the ranging pitch of the sounds of the female. In other words, the want of definite and fixed pitch to the female's song demands for the receiving apparatus of her sounds a corresponding range of co-vibration, so that instead of indicating a high order of auditory development it is really the lowest, except in its power of determining the direction of a sonorous center, in which respect it surpasses by far our own ear.†

* The shafts of the antennæ include an angle of about 40°. The basal fibrils of the antennæ form an angle of about 90°, and the terminal fibrils an angle of about 30°, with the axis of the insect.

† Some physiologists, attempting to explain the function of the semicircular canals, assume, because these canals are in three planes at right angles to each other, that they serve to fix in space a sonorous center, just as the geometrician by his three coördinate planes determines the position of a point in space. But this assumption is fanciful and entirely devoid of reason; for the semicircular canals are always in

The auditory apparatus we have just described does not in the least confirm Helmholtz's hypothesis of the functions of the organ of Corti; for the supposed power of that organ to decompose a sonorous sensation depends upon the existence of an auditory nerve differentiated as highly as the co-vibrating apparatus, and in the case of the mosquito there is no known anatomical basis for such an opinion. In other words, my researches show external co-vibrating organs whose functions replace those of the tympanic membrane and chain of ossicles in receiving and transmitting vibrations; while Helmholtz's discoveries point to the existence of internal co-vibrating organs which have no analogy to those of the mosquito, because the functions of the former are not to receive and transmit vibrations to the sensory apparatus of the ear, but to give the sensation of pitch and to decompose a composite sonorous sensation into its elements; and this they can only do by their connection with a nervous development whose parts are as numerous as those of the co-vibrating mechanism. Now as such a nervous organization does not exist in insects, it follows that neither anatomical nor functional relations exist between the co-vibrating fibrils on the antennæ and the co-vibrating rods in the organ of Corti, and therefore, that neither Hensen's observations on the *Mysis* (assumed by Helmholtz to confirm his hypothesis), nor mine on the mosquito, can be adduced in support of Helmholtz's hypothesis of audition.*

The above described experiments were made with care, and I think that I am authorized to hold the opinion that I have established a physical connection existing between the sounds emitted by the female and the co-vibrations of the antennal fibrillæ of the male mosquito; but only a well established physiological relation between these co-vibrating parts of the animal and the development of its nervous system will authorize us to state that these are really the auditory organs of the insect. At this stage of the investigation I began a search through the zoological journals, and found nearly all that I could desire in a paper, in vol. iii, 1855, of the Quarterly Journal of the Micro-

the same dynamic relation to the tympanic membrane, which receives the vibration to be transmitted always in one way through the ossicles to the inner ear. Really, we determine the direction of a sound by the difference in the intensities of the effects produced in the two ears, and this determination is aided by the form of the outer ear and by the fact that man can turn his head around a vertical axis. Other mammalia, however, have the power of facilitating the determination of motion by moving the axis of their outer ears into different directions. It is also a fact that when one ear is slightly deaf, that the person unconsciously so affected always supposes a sound to come from the side on which is his good ear.

* Also, the organ of Corti having disappeared in the lower vertebrates, it is not likely that it would reappear in the articulata; and especially will this opinion have weight when we consider that the peculiar function of the organ of Corti is the appreciation of those composite sounds, whose signification mammals are constantly called upon to interpret.

scopical Society, entitled "*Auditory Apparatus of the Culex Mosquito*, by Christopher Johnson, M.D., Baltimore, U. S."

In this excellent paper I found clear statements showing that its talented author had surmised the existence of some of the physical facts which my experiments and observations have confirmed. To show that anatomical facts conform to the hypothesis that the antennal fibrils are the auditory organs of the mosquito,* I cannot do better than quote the following from Dr. Johnson's paper:

"While bearing in mind the difference between *feeling a noise* and *perceiving a vibration*, we may safely assume with Carus—for a great number of insects, at least,—that whenever true auditory organs are developed in them, their seat is to be found in the neighborhood of the *antennæ*. That these parts themselves are, in some instances, concerned in collecting and transmitting sonorous vibrations, we hold as established by the observations we have made, particularly upon the *Culex mosquito;* while we believe, as Newport has asserted in general terms, that they serve also as tactile organs.

2.

"The male mosquito differs considerably, as is well known, from the female; his body being smaller and of a darker color,

* A short time before the death of my friend, Prof. Agassiz, he wrote me these words: "I can hardly express my delight at reading your letter. I feel you have hit upon one of the most fertile mines for the elucidation of a problem which to this day is a puzzle to naturalists, the seat of the organ of hearing in Articulates."

and his head furnished with *antennæ* and *palpi* in a state of greater development. (Fig. 2.) Notwithstanding the fitness of his organs for predatory purposes, he is timid, seldom entering dwellings or annoying man, but restricts himself to damp and foul places, especially sinks and privies. The female, on the other hand, gives greater extension to her flight, and attacking our race, is the occasion of no inconsiderable disturbance and vexation during the summer and autumn months.

"The head of the male mosquito, about 0·67 mm. wide, is provided with lunate eyes, between which in front superiorly are found two pyriform capsules nearly touching each other, and having implanted into them the very remarkable antennæ.

"The capsule, measuring about 0·21 mm., is composed of a horny substance, and is attached posteriorly by its pedicle, while anteriorly it rests upon a horny ring, united with its fellow by a transverse fenestrated band, and to which it is joined by a thin elastic membrane. Externally it has a rounded form, but internally it resembles a certain sort of lamp shade with a constriction near its middle; and between this inner cup and outer globe there exists a space, except at the bottom or proximal end, where both are united.

"The antennæ are of nearly equal length in the male and the female.

"In the male, the antenna is about 1·75 mm. in length, and consists of fourteen joints, twelve short and nearly equal, and two long and equal terminal ones, the latter measuring (together) 0·70 mm. Each of the shorter joints has a fenestrated skeleton with an external investment, and terminates simply posteriorly, but is encircled anteriorly with about forty *papillæ*, upon which are implanted long and stiff hairs, the proximal sets being about 0·79 mm. and the distal ones 0·70 mm. in length; and it is beset with minute bristles in front of each whorl.

"The two last joints have each a whorl of about twenty short hairs near the base.

"In the female the joints are nearly equal, number but thirteen, and have each a whorl of about a dozen small hairs around the base. Here, as well as in the male, the parts of the antennæ enjoy a limited motion upon each other, except the basal joint, which, being fixed, moves with the capsule upon which it is implanted.

"The space between the inner and outer walls of the capsule, which we term confidently the auditory capsule,* is filled with a fluid of moderate consistency, opalescent and containing minute spherical corpuscles, and which probably bears the same relation to the nerve as does the lymph in the scalæ of the cochlea of higher animals. The nerve itself, of the antenna, proceeds from the first or cerebral ganglion, advances toward

* See fig. 2.

the pedicle of the capsule in company with the large trachea, which sends its ramifications throughout the entire apparatus, and, penetrating the pedicle, its filaments divide into two portions. The central threads continue forward into the antenna, and are lost there; the peripheral ones, on the contrary, radiate outward in every direction, enter the capsular space, and are lodged there for more than half their length in *sulci* wrought in the inner wall or cup of the capsule.

"In the female the disposition of parts is observed to be nearly the same, excepting that the capsule is smaller, and that the last distal antennal joint is rudimental.

"The proboscis does not differ materially in the two sexes; but the palpi, although consisting in both instances of the same number of pieces, are very unlike. In the female they are extremely short, but in the male attain the length of 2·73 mm.; while the proboscis measures but 2·16 mm. They are curved upward at the extremity.

"* * * The position of the capsules strikes us as extremely favorable for the performance of the function which we assign to them; besides which there present themselves in the same light the anatomical arrangement of the capsules, the disposition and lodgment of the nerves, the fitness of the expanded whorls for receiving, and of the jointed antennæ fixed by the immoveable basal joint for transmitting, vibrations created by sonorous undulations. The intra-capsular fluid is impressed by the shock, the expanded nerve appreciates the effect of the sound, by the quantity of the impression; of the pitch, or quality by the consonance of particular whorls of stiff hairs, according to their lengths; and of the direction in which the undulations travel, by the manner in which they strike upon the antennæ, or may be made to meet either antenna in consequence of an opposite movement of that part.

"That the male should be endowed with superior acuteness of the sense of hearing, appears from the fact, that he must seek the female for sexual union either in the dim twilight or in the dark night, when nothing but her sharp humming noise can serve him as a guide. The necessity for an equal perfection of hearing does not exist in the female; and, accordingly, we find that the organs of the one attain a development which the others never reach. In these views we believe ourselves to be borne out by direct experiment, in connection with which we may allude to the greater difficulty of catching the male mosquito.

"In the course of our observations we have arrived at the conclusion, that the antennæ serve to a considerable extent as organs of touch in the female; for the palpi are extremely short, while the antennæ are very moveable, and nearly equal

the proboscis in length. In the male, however, the length and perfect development of the palpi would lead us to look for the seat of the tactile sense elsewhere, and, in fact, we find the two apical antennal joints to be long, moveable, and comparatively free from hairs; and the relative motion of the remaining joints very much more limited."

My experiments on the mosquito began late in the fall, and therefore I was not able to extend them to other insects. This spring I purpose to resume the research, and will experiment especially on those orthoptera and hemiptera which voluntarily emit distinct and characteristic sounds.

4. *Suggestions as to the function of the Spiral Scalæ of the Cochlea, leading to an Hypothesis of the Mechanism of Audition.*

As the auditory nerve has by far its highest development in the cochlea, it is a natural inference that this part of the ear is chiefly concerned in audition, and that the very peculiar form of the cochlea fulfills some important function; yet the relations of this form to the mode of audition has occupied but little the attention of physiologists. The only suggestion as to the uses of its form with which I am acquainted, is that given by Dr. J. W. Draper in his Physiology, N. Y., 1855. This distinguished scientist states that "it may be imagined how it is that a sound passing through the auditory canal, the bones of the tympanum, the membrane of the fenestra ovalis, and thus affecting its destined portion of the lamina, does not give rise to an idea in the mind of repetition or reverberation by moving back and forth through the two scalæ and affecting its proper nerve fibril at each passage. Is there not a necessity for the exertion of some mechanism of interference which shall destroy the wave after it has once done its work?" Dr. Draper then reasons that this reverberation is prevented by the scalæ being of different lengths and by the fact of their junction in the helicotrema. These two circumstances give rise to interferences, in the helicotrema, of the waves which have proceeded from the stapes up the scala vestibuli with the waves which passed from the membrana tympani across the tympanum to the fenestra rotunda, and thence up the scala tympani to the helicotrema. Dr. Draper also states that when the stapes is pushed in by the contractions of the tensor-tympani and stapedius muscle, the relative length of the scalæ is changed, and thus the proper adjustment for an interference is effected. But even granting that "reverberations or repercussions" take place in a body like the whole apparatus of audition, whose heterogeneous structure must make all of its vibrations, *taken as a mass*, forced oscillations, I do not agree with my distinguished friend in thinking that the difference in the lengths of

the scalæ could bring about any interference except of the most minute and inefficient amount; even if we could agree with Dr. Draper that the intensity of the pulses sent from the fenestra rotunda nearly equal the intensity of those sent up the scala vestibuli from the stapes. The following considerations will make clear our objections to the hypothesis of Dr. Draper. If we take the mean wave-length of the sounds which fall upon the ear as that of the treble G of 440 vibrations per second, it follows that this wave-length will be one meter. But the velocity of sound in the fluid of the scalæ is, at least, $4\frac{1}{4}$ times what it is in air of the same temperature; therefore the average length of the sonorous waves which traverse the scalæ is $4\frac{1}{4}$ meters, and hence for two such waves, meeting in the helicotrema, to completely interfere, one scala would have to exceed the other in length by 2·12 meters. But the entire length of a scala is at the highest only 29 mm., and the difference in their length, taken at its maximum, is so slight that the diminution in the intensity of the resultant wave produced in the helicotrema is inappreciable; and especially will it be so considered when we take into account the relatively feeble intensity of the wave which is sent from the tympanic membrane across the air of the drum on to the membrane of the fenestra rotunda, where two sudden changes in density occur before it passes up the scala tympani.

The following attempt at an explanation of the functions of the spiral stairways of the cochlea is given merely as a suggestion, and with the hope that I may thereby call the attention of students of physiological acoustics to the consideration of the uses of these peculiar forms. Recent studies in embryology and comparative anatomy have shown that the ductus cochlearis is the essential part of the ear, and that the forms of the scalæ are determined by it; for "the original soft parts of the cochlea are distinct from their osseous capsule, which belongs to the petrous bone; the scalæ are secondary formations around the principal canal of the cochlea, the ductus cochlearis, whose epithelial lining proves eventually to be the germ center, so to speak, of the entire apparatus." (*Waldeyer, On the Auditory Nerve and Cochlea;* in Stricker's Histology.) The fact that the ductus controls the form of the scalæ, and not *vice versa*, shows that the scalæ must bear some very important functional relation to the ductus. This relation will become evident on considering the actions which must take place when a sound-wave traverses the scalæ.

All know that the organ of Corti is enclosed in the ductus cochlearis, a canal of triangular section bounded on two of its sides by the scalæ, and on its third by the membranes lining the outer wall of the cochlea. The upper wall of this canal is

formed by the membrana Reissneri, which separates it from the scala vestibuli, and its lower wall is the lamina spiralis, and the elastic membrana basilaris, which separate it from the scala tympani. The ductus is closed at its upper end, and at its lower end it communicates with the sacculus hemisphericus by a fine duct. The arch of Corti rests upon the membrana basilaris, which extends beyond the base of the arch to the membranous outer wall of the cochlea, and over the arch spreads the membrana tectoria, covering the rods of Corti and the hair-cell cords as with a roof, but leaving the outer portion of the elastic membrana basilaris exposed. We will now show that the significance of these anatomical relations is to bring the sound vibrations to act with the greatest advantage on the co-vibrating parts of the ear, and to cause these parts to make one-half as many vibrations in a given time as the tympanic or basilar membranes.

The relations which the form of the scalæ bears to the sonorous waves traversing them, will be modified according to the existence or non-existence of a communication between the scalæ. On this point there seems to be some difference of opinion, and, therefore, I will attempt to explain the functions of the scalæ, first, on the supposition that they are continuous, and then on the assumption that they are not continuous, but closed at the place where the helicotrema is supposed by most anatomists to exist.

E. Weber was the first to point out the peculiar molecular actions which exist when the dimensions of a body are very small compared with the length of the sonorous waves which traverse it; and Helmholtz based his investigations on *the Mechanism of the Ossicles of the Ear* on the theory of Weber, which Helmholtz gives in these words: "The difference in displacement of two oscillating particles, whose distance from one another is infinitely small compared with the wave-length, is itself infinitely small compared with the entire amplitude of displacement." It is evident that the compressions and dilatations which may exist in any body, depend entirely on the differences in the phases of the vibrations constituting the sonorous wave, and when the body has a depth equal to half a wave-length it can embrace the maximum amount of condensation and rarefaction. But condensation and dilatation alone produce *lateral action* on the walls of a straight canal traversed by sonorous vibrations, and hence, if the length of the canal be but a small fraction of the wave, then there exists throughout the canal but little difference in phase, and therefore but little lateral action. Now the united lengths of the scalæ is but a small fraction of the mean length of the sonorous waves which traverse it; for if we take, as above, $4\frac{1}{2}$ meters as the mean

length of the waves which are propagated through the scalæ, and 59 mm. as the length of the united scalæ, it follows that the latter is only $\frac{1}{72}$ of the mean wave-length. Now if we imagine the scalæ straightened and forming one continuous tube with a free communication existing at the helicotrema, then the mean wave traversing them will cause only $\frac{1}{22}$ of the lateral action which this same wave would produce if the scalæ had the length of half of the wave, and it follows that the whole liquid of the scalæ would vibrate forward and backward almost as an incompressible mass, approaching in character to the oscillations of a solid piston in a cylinder; therefore, the action against the walls of the ductus cochlearis would be very slight. But now consider the change in effect on the ductus which takes place when it, together with the scalæ, is wound up into such an ascending spiral as exists in the ear. The molecules of the liquid in the scalæ, thrown forward and backward by the vibrations of the stapes, tend to move in straight lines, but the now curved form of the scalæ causes them to press against the outer or peripheral part of the upper wall (membrana Reissneri) of the ductus cochlearis, and against the outer part of the lower wall (membrana basilaris) when the stapes moves inward, and when it moves outward this action of compression is relieved from the two opposite walls of the ductus. But these actions produced by the stapes on the two walls of the ductus are opposed to each other, and since they take place simultaneously and with about the same intensity, (by reason of our assumption of the free communication of the scalæ,) the rods of Corti and the hair-cells will not vibrate but will only experience compressions and dilatations like the fluid in which they are immersed. Therefore, there appears to me a physical basis for the opinion that either there is no communication between the scalæ, or, if the helicotrema exist, that it must be a very constricted passage. Indeed, if we adopt the latter view, then everything works to produce the maximum effect on the co-vibrating parts of the organ of Corti; for when the stapes moves inward the pressure is thrown on the outer border of the upper wall, or roof, of the ductus, thence across to the peripheral portion of the basilar membrane. This action, we may say, takes place simultaneously throughout the whole length of the ductus, moves downward the floor of the basilar membrane, and thus presses the fluid of the scala tympani against the fenestra rotunda and moves this membrane outward. When, however, the stapes moves outward, the pressure is relieved from the elastic basilar membrane, which is now moved upward, while the fenestra rotunda moves inward.*

* If we could examine, at the same time, vibrating points on the stapes and on the fenestra rotunda with a vibration-microscope, I imagine that these points would exhibit no difference in phase when the membrana tympani vibrated to a note below the treble.

There are also other anatomical facts besides the inclination of the membrana Reissneri to the plane of the membrana basilaris, and the inclination of both these membranes to the plane perpendicular to the axis of the cochlea, which favor an opinion that the outer or peripheral part of the basilar membrane receives the main part of the vibrations which enter the ductus. The auditory nerve fibrils are not attached to the Corti rods or pillars, as was formerly imagined; and, therefore, these bodies cannot be the co-vibrating parts of the ductus; but the Corti pillars appear to act, in conjunction with the cylindrical nerve-cells of Hensen, as supports for the lamina reticularis, between which and the basilar membrane are steadily and tensely stretched the hair-cell *cords* (as I will term them); and to these cords are attached the nerve-fibrils. Waldeyer says, on this point, that "The outer radial fibers direct their course, as Gottstein has found, toward the tunnel of Corti, passing between the inner pillars and traversing the tunnel about midway between the summit and base of the arch; in a profile view these fibers appear like stretched harp-springs. On leaving the arched space they pass between the outer pillars and direct their course—rising a little toward the scala vestibuli—straight to the hair-cells, with which they become completely fused. In several preparations from the dog and the bat I have seen this termination of the nerves in the most convincing manner, at least so far as the innermost row of hair cells is concerned; as to the other rows, we may pretty confidently assert that the termination of the nerves is the same, for we can frequently see several fibers passing at the same time between the outer pillars." The very fact that the number of these hair-cell cords increases with the higher development of the ear shows their important function; for, while in man they are arranged alternately in five rows and number 18,000, in other Mammalia there are only two or three rows.* These hair-cell cords are more perpendicular to the basilar membrane than the Corti rods, and are also different in their forms, having swellings in the middle of their lengths. These swellings must cause them to act like loaded strings, and each hair-cell cord is peculiarly well adapted to co-vibrate with only one special sound. Also, these hair-cell cords are placed in reference to the sound pulses, striking them somewhat in the relation which the antennal fibrils of the mosquito bear to a wave-surface to which their lengths are per-

* It is to be regretted that no accurate measures of the lengths and diameters of the rods and cords of the organ of Corti have been secured. The outer pillars of the arch of Corti certainly double their length in going from the base to the top of the ductus; but does this fact point them out as bodies suitably proportioned to co-vibrate to sounds extending through at least *eight octaves?* I known of no measures on the hair-cell cords. When their dimensions are determined, physiologists will be able to give more precision to their hypotheses.

pendicular. The hair-cell cords, therefore, will not be set in vibration by the action of the feeble pulses which may reach them directly through the membrana Reissneri from the scala vestibuli; and furthermore, the shielding influence of the membrana tectoria tends to prevent this direct action on the cords.

If my view be correct, that these cords receive their vibrations from the basilar membrane, and not directly from the impulses sent into the ductus, it necessarily follows that these cords bear to the membrane, to which they are attached, the same relation as stretched strings bear to the vibrating tuning-forks in Melde's experiments; and, therefore, *a cord in the ductus will vibrate only half as often in a second as the basilar membrane to which it is fastened.* Experiments, similar to those described in section 1 of this paper, illustrate very well our hypothesis of audition. Thus, the membrane, placed near the sounding reed, stands for the basilar membrane; strings, of various lengths and diameters and loaded at their centers, are fastened to the membrane and represent the hair-cell cords. On sounding the reed-pipe, only those strings in tune with the harmonics existing in the composite sound of the reed will enter into vibration; just as when the same sound vibrations enter the ear, and vibrate the basilar membrane, the only hair-cell cords which enter into vibration are those in tune with the elementary vibrations existing in the membrane. Also, it is to be observed that as the loaded string makes one vibration to two of the membrane, so the hair-cell cord makes only one vibration to two of the basilar membrane.

If it be true that when simple vibrations impinge on the ear, the tympanic and basilar membranes vibrate twice, while the co-vibrating body only vibrates once, then it follows that if the same simple vibrations can be sent directly to the co-vibrating parts of the ear, without the intervention of the basilar membrane, we should perceive a sound which is the octave of the one we experienced when the same simple vibrations entered the ear through the tympanic membrane. Hence it appears that our hypothesis can be brought to the test of experiment in the following manner: A tuning-fork held near the ear causes a sensation corresponding to the designated pitch of the fork. But the vibrations of this fork can be sent to the inner ear through the bones of the head; and although we cannot prevent the simultaneous vibration of the tympanic and basilar membranes, yet we can at the same time directly vibrate all the parts of the inner ear. Therefore, if we first hold this fork near the ear and note its pitch and the quality of its sound, and then press its foot firmly against the temporal bone, we should perceive a marked difference in the timbre of the fork when sounded in these two different positions; for when its foot is

against the head we should hear the usual simple sound of the fork accompanied by its octave.

Thus, if we take an $Ut_6$ fork and vibrate it near the ear, and closely apprehend the character of its sound, we shall experience a sensation which certainly does not contain that corresponding to the higher octave of the fork. Now, press firmly the foot of the fork against the zygomatic process, close to the ear, directing the foot of the fork somewhat backward, and we shall distinctly hear the higher octave of the fork singing in concert with its real note. If the auditory canal be now closed by gently placing the tip of the finger over it, we shall perceive the higher octave with an intensity almost equal to that of the fundamental note. The same sensation, though less intense, may be obtained by placing the fork on any part of the temporal bone. One can also perceive distinctly the higher octave when the fork is placed on the parietal bone, about two inches in front of and an inch or so to the side of the foramen, and its foot directed toward the opposite inner ear, while the auditory canal of this ear is gently closed with the tip of the finger. But the higher octave sings out with the greatest intensity when the foot of the fork is placed on the tragus of the outer ear. A friend, who is a musician as well as a physicist, repeated these experiments, and he informs me that when the foot of the fork is placed against the tragus of his ear he hears the higher octave to the almost entire exclusion of the lower, and with a clearness that reminds him of the sensation perceived when an $Ut_4$ resonator, placed to the ear, reinforces its proper note. The higher octaves of several forks have been thus perceived, but the forks from $Ut_3$ to $Ut_4$ inclusive appear to give the best results.

The fact that sound pulses sent to the inner ear through the head give the sensation corresponding to the higher octave of that perceived when the fork vibrates the air outside the ear, and, therefore, that different co-vibrating parts of the ear are set in action by the vibrations reaching the ear by these two different routes, is a necessary consequence of my hypothesis of the mode of audition, and was not suspected until my hypothesis pointed it out to me, and was not known until I attempted to test the hypothesis by experiment. I know of no other hypothesis that accounts for this fact, which, while it is a necessary consequence of my own views, is directly opposed to those hypotheses hitherto formed on the mode of audition ; for, according to the latter, the co-vibrating parts of the ear make as many oscillations in a given interval as the tympanic and basilar membranes.

[To be continued.]

## 5. *Six Experimental Methods of Sonorous Analysis described and discussed.*

THE remarkable discoveries in sound made in these later times by Helmholtz were owing, in great part, to his having early seen the necessity of obtaining precise knowledge of the composition of sounds,—by means of the methods of sonorous analysis which he had devised,—before one could attempt to give an explanation of the causes of timbre, of the mechanism of audition, and of the physiological causes of musical harmony; and furthermore, he reduced all of the analytic methods and explanations, contained in his classical work, to a harmonious system, by showing how they naturally flowed from the fertile theorem of Fourier. As Helmholtz distinctly states: "In letzter Instanz ist also der Grund der von Pythagoras aufgefundenen rationellen Verhältnisse in dem Satze von Fourier zu finden, und in gewissem Sinne ist diese Satz als die Urquelle des Generalbasses zu betrachten." (Tonempfindungen, p. 346.)

The evident importance of the subject of sonorous analysis will probably render interesting the few remarks I here venture to offer on this subject. I will describe in order six meth-

ods of sonorous analysis. Methods 1, 2 and 5 have been used by Helmholtz and König. Methods 3, 4 and 6, as far as I know, originated with me. To render comparable all of the experiments which I shall describe, I shall always use one composite sound of uniform intensity by sounding with a blast of constant pressure an $Ut_2$ Grenié free-reed pipe, from which has been removed its reinforcing pyramidal pipe.

(1.) *Analysis by means of Resonators applied directly to the ear.*

This method of Helmholtz is so well known that it need not here be described, but I will give some experiments which I have made on its degree of precision under the head of the sixth method of analysis, to be described.

(2.) *Analysis by means of Resonators connected with König's manometric flames.*

This is the least delicate and accurate of all methods of sonorous analysis, but it has a value in giving an objective, ocular illustration, which is sometimes of use. I do not, however, here refer to the use of a manometric flame placed in connection with a conical pipe, into which one sings, and which instrument in the hands of König has done such admirable work in the analysis of the vowel sounds; but I refer to the harmonic series of resonators connected with as many flames which burn near a long revolving mirror.

The following experiments will show the degree of delicacy of the above apparatus. I sounded the reed pipe with its maximum intensity when it was about one foot from the center of the series of resonators, and produced well marked serrations in the $Ut_3$ and $Ut_4$ flames, but the other flames showed only slightly indented top borders. It was only when I sang loudly $Ut_3$ or sounded this same note on a French horn that the edges of the flames became deeply serrated.

(3.) *Analysis by means of Resonators which are successively brought near the source of origin of a composite sound and thus successively reinforce all of its sonorous elements.*

If we take any two resonators separated by a known interval, and vibrate before them the forks of this interval, we can carry these sounds in the memory. Now close the nipples of the resonators with wax and successively bring their mouths near the origin of the composite sound. If the simple sounds to which the resonators are tuned exist as components of the sound, we shall hear them singing out clearly above the general chorus of the other harmonics. Thus I have often successfully shown to an audience the composition of a composite sound.

(4.) *Analysis by means of Resonant boxes carrying solid bodies tuned in unison with the sounds to which the air in these boxes resounds.*

This method is an excellent one when the composite sound can be obtained with intensity, when the boxes are accurately in tune with the solid bodies (forks or strings) attached to them, and when the boxes are in unison with the sonorous elements of the composite sound which we would analyze. The last mentioned condition is not always fulfilled in the boxes on which forks have been sometime mounted, for the former are apt to change their interior capacity by warping. This fact can readily be ascertained by partly introducing the hand into the mouth of the box, and noting the effect on the intensity of the sound.

This method of analysis is similar to the one previously described; for the resonant box of a fork acts like a resonator and can be used to intensify any harmonic of a composite sound; but there is an important difference in the methods, for the fork or the string being in unison with the proper note of the mass of air in the box, is set in vibration by the latter, so that after the box has been removed from the vicinity of the origin of the composite sound, and the latter has ceased, we find that the fork sings out alone, and thus shows that it has selected from a chorus of harmonics that one which is in unison with its own tone. I have thus been able, by placing one fork after another, of the series of the harmonics of the $Ut_2$ reed, to show the composition of its sound to a large audience with entire satisfaction. I have also succeeded with the following experiment. Forcibly sound the reed, and place around the opening of its "stump" all of the eight forks, of the harmonic series of $Ut_2$, with the mouths of their resonant boxes toward the reed. After the reed has sounded for a few seconds, stop it, and we shall find that all of the forks are in vibration; and thus singing together, they approximately reproduce the sound of the reed. This experiment to succeed requires the resonant boxes, the forks, and the harmonics of the reed to be in exquisite unison.

(5.) *Analysis by means of the beating of simple sounds of known pitch with the harmonics of the composite sound to be analyzed.*

If we use forks for the simple sounds, it will be better slightly to flatten or sharpen the note of the sound to be analyzed. Then knowing the number of beats that the fundamental of the note gives with its corresponding fork, we can designate the number of any other harmonic by the number of beats it makes with a fork of known pitch; for the number of beats observed, when referred to the number of beats of the fundamental as unity, will be directly as the number of the harmonic

in the series. If we cannot well alter the pitch of the composite sound, then it will be well to use forks with sliding weights on graduated prongs, or loaded strings accurately tuned and provided with meter scales. In this system of analysis it is very important to guard against being led astray by the beating of resultant tones; therefore, the forks or strings should be gently vibrated and resonators used to assist the ear.

(6.) *Analysis by means of a loose membrane which receives the composite sonorous wave and transmits its vibrations through filaments or light rods to a series of forks mounted on their resonant cases.*

This method of analysis is the one we devised in our experimental confirmation of Fourier's theorem and described in sec. 1 (pp. 82–84). We here wish to call attention to the precision of this method of analysis, while at the same time acknowledging the delicate instrumental conditions required to use it successfully. The principal interest attached to it is that it shows in a vivid manner the operation of the instantaneous decomposition of a composite wave into its elementary pendulum-vibrations. The method has also peculiar interest as showing, in the most striking manner, the exaltation of the action of very feeble periodic impulses to such degrees of intensity as to set into synchronous vibration very large masses of matter; and it may be well before we discuss the subject proper of this section to call attention to this very interesting result, as set forth in the following experiment with our apparatus. To the membrane, covering the hole in the box of the reed-pipe, I attached one end of a fiber of silk-worm cocoon, one meter long and weighing one milligram. The other end of the fiber was cemented to the face of a prong of an $Ut_2$ fork. This fork weighed 1,500 grams, while the top of its resonant box and the air in the latter weighed respectively 102 and 22 grams. Therefore, the fiber set in motion 1,624,000 times its own weight by only a fraction of the force which traversed it, and even this force was a yet more minute fraction of the whole energy of the aerial vibrations produced by the reed.

An experiment like the above is instructive as an analogical illustration of the manner in which we may imagine an ætherial vibration to produce chemical decomposition by causing such powerful synchronous vibrations in the molecules of compounds as to shake their atoms asunder; and we have already seen how very feeble impulses sent through a medium of great tenuity can, when rapidly recurring and of the proper period, produce mechanical effects which at first sight appear incredible. Time is required in both cases to produce an appreciable action. The time required in the case of the sonorous vibrations de-

creases as their number per second increases, and in the case of the ætherial vibrations we have analogical phenomena. In the acoustical experiment, if the fork be much out of tune with the pulses transmitted by the fiber, no motion is produced in the fork; likewise, we may imagine that when the period of vibration of one or more of the constituent atoms of a certain molecule is far removed from unison with any of the ætherial vibrations falling upon it, no motion, or chemical decomposition, will ensue.

The analogy between the two classes of phenomena is yet more striking when we remember that the fork *selects*, from the composite vibratory motion which traverses the fiber, only that vibration which is in unison (or only slightly removed from unison) with its own proper periodic motion; so, likewise the molecule, or atoms of the molecule, select only those vibrations from the ray which are in unison with their own atomic periods; and on the tuning of the atom depends whether the result of the action of the ray will evince itself as heat, phosphorescence, chemical action, or fluorescence.

The following experiments show that the method of analysis we are now discussing surpasses in delicacy and sharpness of definition any other method in which sympathetically vibrating bodies are employed. As already shown, the forks select from the composite vibratory motion which strikes them only those simple vibrations which are in unison with their own vibratory periods. This remark, however, requires some modification, though the qualification necessary is less than is required when other similar methods of analysis are used. In all cases of co-vibration there is a certain range of pitch, above and below the sound which is in unison with the existing vibration, through which the co-vibrating body responds. The farther the remove from unison the weaker the response.* But in some cases a slight remove of the pitch from unison will cause a great diminution in the intensity of the co-vibrations, while in others the same departure from unison causes only a slight or even inappreciable change in their intensity. For example, I connected the $Ut_3$ fork—the second in the harmonic series of $Ut_2$—to the membrane on the $Ut_2$ reed, and sounded the latter during a few seconds. After the reed was silent, I heard the fork sounding with intensity.† But, on loading the fork with a

* See Helmholtz's Tonempfindungen, p. 217, for the law connecting the variation of intensity of co-vibration with the variation of pitch in the existing body.

† It is perhaps hardly necessary to state that in all of these experiments I first ascertained that when the fiber was detached from the fork, the latter did not emit a sound caused by the action of the intervening vibrating air. This condition is readily attained by screening the mouths of the resonant boxes, or by turning these mouths away from the sounding reed. Also, I should here state that the reed was tuned to the fork after the fiber was stretched between the latter and the membrane on the reed-pipe.

piece of wax, so that it gave *five* beats per second with the note of the fork when unloaded, I could not, by any variation in the tension of the fiber or of any other circumstances of the experiment, set in vibration the fork by means of the pulses sent from the reed through the fiber; yet, on placing the nipple of an $Ut_3$ resonator in my ear, I perceived that this flattened note of the fork produced a decided resonance, thus showing that although the fork could not respond to its own note flattened five beats per second, yet the resonator, under the same circumstances, did enter into sympathetic vibration. When the fork gave *four* beats per second it responded to the reed, but this response was only audible on placing the ear close to the mouth of the fork's resonant box. With *three* beats per second the sound of the fork was readily perceptible, while the resonator reinforced it very decidedly. When the fork was out of unison *two* beats per second, its sound was slightly increased; and with a departure of *one* beat per second, the response of the fork was yet stronger, but greatly inferior in intensity to that produced when the fork was in unison with its proper sound—the second harmonic of the reed; yet the resonator reinforces this flattened sound as forcibly as it does that which emanates from the unloaded fork. These facts concerning the want of sharpness in the detection of pitch by means of resonators are not in accordance with the statements made in recent popular works on sound, where the resonator is described as remaining dumb until the exact pitch to which it is tuned is reached, when it responds with a suddenness which has been compared to an explosion!

The fork, the stretched fiber and the intensity of the sound of the reed remaining in the same conditions as in the above experiments, I gradually unloaded the fork until it made only *one beat in eight seconds*, and yet even this slight departure from unison with the second harmonic of the reed was evident in the difference in the intensities of the fork's responses when thus loaded and when the wax was removed. This fact I have repeatedly confirmed by testing the intensities of the two sounds by different hearers, who were placed so that they could not see when the fork was loaded or unloaded. Now E. H. Weber has found that only the most accomplished musical ears can distinguish between the pitch of two notes whose vibrations are as 1000 to 1001, but by the above method we can readily detect a departure from unison in the two notes amounting to the interval of 2000 to 2001, or to the $\frac{1}{120}$th of a semitone.

In connection with the preceding observations, the following experiments on resonators and sympathetic vibrations may be of interest. I substituted for the ear the manometric flames of König, viewed in a revolving mirror, and tested the response of

the resonators to sounds not in accord with their proper notes. The results agreed with those previously obtained on placing the resonator to the ear. I now mounted an $Ut_3$ fork on its resonant case, and sounding it *strongly* before all the resonators of the harmonic series of $Ut_2$, I caused all of the manometric flames connected with these resonators to vibrate, each giving the same number of serrations as when the $Ut_3$ fork was brought near its own resonator. The same result was obtained when the fork was separated from its case, with this important difference, however, that when the face of fork $Ut_3$ was brought near the mouth of the $Ut_4$ resonator, the flame connected with this resonator gave at the same time serrations belonging to both $Ut_3$ and to $Ut_4$, and this result accords with the following experiment. If one sounds the $Ut_3$ fork on its box, and brings the mouth of the latter near the mouth of the box of the $Ut_4$ fork, the $Ut_4$ fork will co-vibrate, and after $Ut_3$ has ceased to vibrate, $Ut_4$ will sound out quite clearly. If, however, $Ut_4$ be lowered in tone, by weighting it with a piece of wax, so that it gives two beats per second with its proper tone, then the $Ut_4$ fork cannot be set into sympathetic vibration by impulses from $Ut_3$. Also, if forks $Ut_3$ and $Ut_4$ be detached from their boxes, and fork $Ut_3$ be strongly vibrated while the face of one of its prongs remains quite close and parallel to the face of a prong of fork $Ut_4$, the latter is set in vibration by the impulses of $Ut_3$ sent through the intervening air. Fork $Ut_3$ was now loaded so that it successively made 1, 2, 3, and 4 beats per second with the true $Ut_3$ pitch. When it made 3 beats per second, it caused $Ut_4$ to vibrate so feebly as to be barely audible, while 4 beats per second departure of $Ut_3$ from unison produced no effect on $Ut_4$.

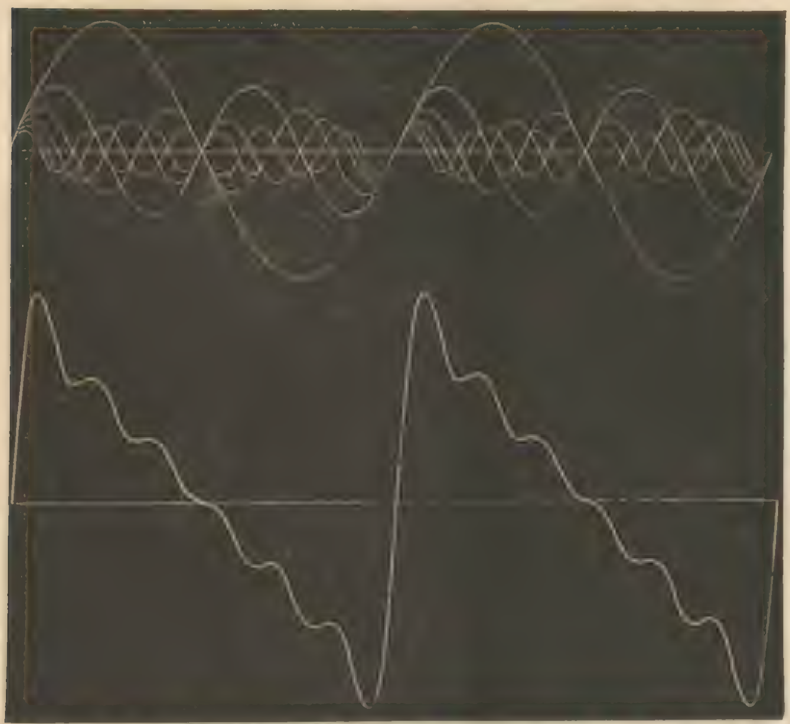
The following experiment was now made to show the want of precision in the determination of the exact pitch of sonorous elements by means of resonators. We have just seen that the $Ut_3$ fork could not vibrate the $Ut_4$ fork when both forks were on their cases, and when $Ut_3$ was flattened by two beats per second: and also that a departure of four beats per second prevented $Ut_3$ from setting $Ut_4$ in sympathetic vibration, when both forks were off their resonant cases, and with their prongs close and parallel to each other. But, when the $Ut_3$ fork was loaded so that it made from 15 to 20 beats with its proper tone, it caused the serrations of the $Ut_3$ resonator to appear, accompanied by the serrations of its octave. The $Mi_3$ fork, although it developed the serrations belonging to its own pitch when brought opposite the $Ut_3$ resonator, yet did not,—as might have been expected,—develop its own tone and the octave when brought near the mouth of the $Ut_4$ resonator. It is probably not necessary to add that the above effects of simul-

taneously developing in the flame the serrations of the proper note of a resonator and those of its octave are only produced when the fundamental sound is intense.

(7.) *The Curve of a Musical Note, formed by combining the sinusoids of its first six harmonics; and the curves formed by combining the curves of musical notes corresponding to various consonant intervals.*

We have already seen that any composite vibration, which produces in us the sensation of a musical note, can always be reproduced by the simultaneous production of a certain number of the simple sounds of a harmonic series, provided these simple sounds have the proper relative intensities. Therefore to obtain the resultant curve corresponding to a musical note, we draw on one axis its harmonic components with their proper wave-lengths and amplitudes, and the algebraic sums of their corresponding ordinates are the ordinates of the required resultant curve.

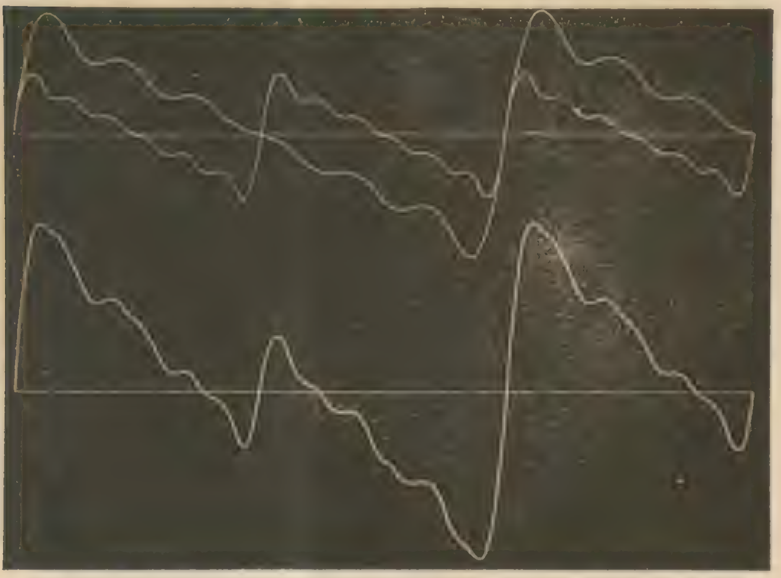
3.



*Curve of a Musical Note; being the resultant of the simple vibrations of its first six harmonics.*

Fig. 3 is the curve of a musical note, being the resultant of the simple vibrations of its first six harmonics. The first six harmonics having been drawn on a common axis, I erected 500 equidistant ordinates, and extended these ordinates some distance below the axis on which I desired to construct the resultant. The algebraic sum of the ordinates, passing through the harmonic curves, were transferred to the corresponding ordinates of the lower axis, and by drawing a continuous line through these points, I formed the resultant curve. The first six harmonics are alone used in the combinations which I have given, because the 7th, 9th, 11th harmonics, and the major number of those above the 12th, form dissonant combinations with the lower and more powerful harmonics. Indeed, the harmonics above the 6th are purposely eliminated from the notes of the piano by striking the string in the neighborhood of its 7th nodal point. The amplitudes of the harmonics of fig. 3 are made to vary as the wave-length; not that this variation represents the general relative intensities in such a composite sound, but they were so made to bring out strongly the characteristic flexures of the resultant. To simplify the consideration of the curves, they are all represented with the same phase of initial vibration. Of course the resultants have an infinite variety of form, depending on the difference in the initial phase, and on the amplitudes of the harmonic elements.

4.



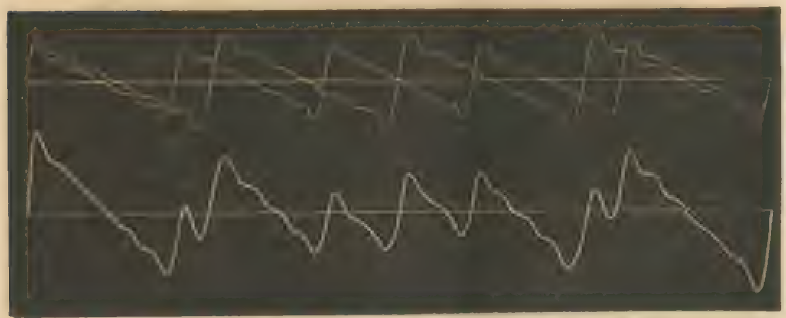
*Resultant curve of a musical note combined with its octave.* $\lambda : \lambda' :: 1 : \frac{1}{2}$.

In figs. 4, 5 and 6, I have drawn the resultant curves formed by combining the curves of musical notes corresponding to the various consonant intervals indicated below the curves. As these curves are the resultants formed by the combination of the composite vibrations of musical notes, it follows that the components of these curves are not simple harmonics, as in the case of fig. 3, but are derived from the *resultant* of fig. 3, by reducing to one-fourth the amplitude of that curve and by taking wave-lengths corresponding to the intervals indicated below the figures.

5.

*Resultant curve of a musical note combined with its fifth.* $\lambda : \lambda' :: 1 : \frac{2}{3}$.

6.



*Resultant curve of a musical note combined with its major third.* $\lambda : \lambda' :: 1 : \frac{4}{5}$.

All of the curves which I have given in this paper were first drawn on a large scale and then reduced by photography to a size suitable to be transferred to the engraver's block.

(8.) *Experiments in which are produced from the above curves (sec. 6) the Motions of a Molecule of Air when it is animated with the resultant action of the six elementary vibrations forming a musical note; or is set in motion by the combined action of sonorous vibrations forming various musical intervals.*

We may imagine the curve corresponding to a musical note, represented in fig. 3, as formed by the trace of a vibrating molecule of air, or of a point of the tympanic membrane, on a surface which moves near these points. Therefore if we slide this curve along its axis, under a slit in a screen which allows only one point of the curve to appear at once, we shall reproduce in this slit the vibratory motion of the aerial molecule and

7.

of the point on the tympanic membrane. I have exhibited this motion in a continuous, or rather, recurring manner, as follows: On a piece of Bristol board I drew a circle, and in one quadrant of this circle I drew 500 equidistant radii. On these radii, as ordinates, I transferred the corresponding values of the same ordinates of the resultant of fig. 3, diminished to one-fourth of their lengths. I thus deflected the axis of curve fig. 3 into one-fourth of a circle curve; and this repeated four times on the Bristol board, rendered the curve continuous and four times recurring, as shown in fig. 7. I now cut this curved figure out

of the board and used it as a template. I placed the latter centered on a glass disc of 20 inches in diameter. The disc was covered on one side with opaque, black varnish, and with the template and the separated points of a pair of spring-dividers, I removed from the glass disc a sinuous band, as shown in fig. 7. The glass disc was now mounted on a horizontal axis and placed in front of a lantern the diameter of whose condensing lens was somewhat greater than the amplitude of the curve. The image of that portion of the curve which was in front of the condenser was now projected on a screen, and then a piece of card board having a narrow slit cut in it was placed close to the disc, with the slit in the direction of one of its radii. On now revolving the disc I reproduced on the screen the vibratory motion of a molecule of air, or of a point on the tympanic membrane, when these are acted on by the joint impulses of the first six harmonic or pendulum vibrations, forming a musical note. On slowly rotating the disc one can readily follow the compound vibratory motion of the spot of light; but on a rapid revolution of the disc, persistence of visual impressions causes the spot to appear lengthened into a band; but this band is not equally illuminated—it has six distinct bright spots in it, beautifully showing the six inflections in the curve.

By sticking a pin in the center of fig. 7, as an axis about which revolves a piece of paper with a fine slit, the reader can gain some idea of the complex motion which I have described.

From the curves of figures 4, 5 and 6 can similarly be reproduced their generating motions. Of course it is understood that in all these cases the amplitude of these vibrations are enormously magnified when compared with the wave-lengths, and that it is really only when the amplitudes of the elementary pendulum vibrations are infinitely small that the resultant curves we have given can be rigorously taken as representing what they purport to; for the law of the "superposition of displacements" depends on the condition that the force with which a molecule returns to its position of equilibrium is directly proportional to the amount of displacement, and this condition only exists in the case of infinitely small displacements; yet the law holds good for the majority of the phenomena of sound.

As a periodic vibration can alone produce in the ear the sensation of sound, and as the duration of the period is always equal to the least common multiple of the periods of the pendulum vibrations of the components, it follows that in the case of a simple sound, or of a sound formed of a harmonic series, that the period equals the time of one vibration of the fundamental; but in the case of any other combinations the duration of the period of the recurring vibration increases with the complexity of the ratio of the times of vibration of the compo-

nents; thus, the durations of the following combinations are placed after them in fractions of a second.

$$C_3+C_4=\tfrac{1}{256};\quad C_3+G_3=\tfrac{1}{128};\quad C_3+E_3=\tfrac{1}{64};\quad C_3+E_3+G_3=\tfrac{1}{25\cdot6};$$
$$C_3+E_3+G_3+C_4=\tfrac{1}{17}\text{th of a second.}$$

The above mentioned facts suggest a curious physiological inquiry, viz: Does it require a combination of sounds, simple or composite, to remain on the ear the duration of an entire period, in order that it shall give the same sensation as is produced when the same combination is sounded continuously? In other words, will a portion of the recurring composite vibration produce the same sensation as an entire period or several periods? The solution of this problem has been the object of a prolonged experimental research, but up to this time the results have been so difficult of interpretation that I have not arrived at any certain knowledge on the subject. I shall, however, return to this interesting but very difficult work.